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ABSTRACT

A general criterion of approximation of boundary layer separation is obtained and from it deductions are made in three examples regarding the conditions under which the point of boundary layer separation is eliminated, and how it is displaced by the effect of a transverse magnetic field.

The general equation for laminar boundary layers of a fluid of high magnetic diffusivity in the presence of a transverse magnetic field is written as follows, using Meksyn's notations (Ref 1):

$$f''' + ff'' = \lambda(1 - f'^2) + \gamma(1 - f') - 2\xi\left(f''\frac{\partial f}{\partial \xi} - f'\frac{\partial f'}{\partial \xi}\right), \tag{1}$$

where

$$\lambda(\xi) = -\frac{d(\mathrm{Log}\, u_0^2)}{d\xi}, \qquad \gamma(\xi) = -\frac{2\sigma B^2 U \xi}{f u_0^2} \tag{2}$$

and with the following conditions at the boundaries

$$f(\xi, 0) = 0, \qquad f'(\xi, 0) = 0, \qquad f'(\xi, \infty) = 1. \tag{3}$$

We propose to study this equation at the point of boundary layer separation as characterized by the following supplementary condition:

$$f''(\xi, 0) = 0. \tag{4}$$

In the case of related boundary layers, it can be said a priori that the criterion of separation consists of a relation between the constants λ and γ. But in the general case there is no reason, strictly speaking, for a general criterion to exist. At the same time, the thickening of the boundary layer in the area of separation causes the real laws $u_0(x)$, $\lambda(\xi)$ and $\gamma(\xi)$, in the absence of separation, to be considerably different from the theoretical ones. The magnetic field, however, reduces the thickness of the boundary layers, this effect being less important in magnetohydrodynamics.

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Since our aim has not been to determine with precision the point of boundary separation, we have gotten around these difficulties by means of approximations, adopting the theoretical law for the law of velocities u_0 (x), and attempting to establish an approximate general criterion.

The saddle-point method applied by Meksyn to integration of boundary layer equations makes it possible to write down the condition at infinity as a function of λ, γ, a = f" (ξ, 0), and of their derivatives.

$$f'(\xi, \infty) = \sum_{n=0}^{\infty} d_n \Gamma\left(\frac{n+1}{4}\right) = 1. \tag{5}$$

By canceling out a, the coefficients d_n with even subscripts disappear, /2 leaving the following:

$$\begin{cases} d_1 = \frac{1}{4}[24(\lambda+\gamma)]^{\frac{1}{2}}, \\ d_3 = -\frac{4}{5}\gamma, \\ d_5 = \left(\frac{24}{\lambda+\gamma}\right)^{\frac{1}{2}}\left[\frac{4-15\lambda}{56}(\lambda+\gamma) + \frac{1}{140}\gamma^2 + \frac{10}{56}(\lambda'+\gamma')\xi\right], \\ d_7 = \frac{24}{175}(3\lambda-1)\gamma + \frac{16}{2{,}625}\frac{\gamma^3}{\lambda+\gamma} + \frac{8}{175}\frac{\gamma(\lambda'+\gamma')\xi}{\lambda+\gamma} - \frac{12}{25}\gamma'\xi + \frac{24}{25}\frac{a'^2\xi^2}{\lambda+\gamma}. \end{cases} \tag{6}$$

Since a' appears from d_7 onwards, it is within the extent to which this development limited to d_5 is fairly complete that it is possible to arrive at an approximate general criterion. In limiting it to d_1, then to d_3, absolutely general successive approximations are obtained:

First approximation

$$\lambda + \gamma = 0.212; \tag{7}$$

Second approximation

$$\lambda + \gamma = 0.212\left(1 + \frac{4}{5}\gamma\right)^2. \tag{8}$$

In order to take into account d_5, where the derivatives λ' and γ' as well as the variable ξ intervene, λ and γ must not be independent functions. We assume then that the magnetic field is constant, and, by taking γ as a new variable, and $u = [24\,(\lambda + \gamma)]^{1/2}$ as a new unknown function, we obtain

$$\frac{1}{\Gamma\left(\frac{4}{5}\right)}\left(1+\frac{4}{5}\gamma\right)=\left(\frac{2}{7}+\frac{15}{112}\gamma\right)u-\frac{5}{896}u^3+\frac{3}{35}\frac{\gamma^2}{u}+\gamma\frac{du}{d\gamma}\left[\frac{5}{672}u^2+\frac{5}{28}(1-\gamma)\right]. \tag{9}$$

The numerical solution of this equation is compared with approximations (7) and (8) in Figure 1. For $-2 \leq \gamma < 0$ there is very close agreement. In this zone it is possible to use the following as the approximate criterion of boundary layer separation in the presence of a uniform transverse magnetic field: /3

$$\lambda + \gamma = 0.2. \tag{10}$$

The following table and Figure 2 and Figure 3 show the conclusions that may be drawn from the three classic examples with a constant transverse magnetic field:

(1) Plane sheet in a linearly retarded flow;

(2) Circular cylinder (radial field);

(3) Plane sheet in a divergent flow.

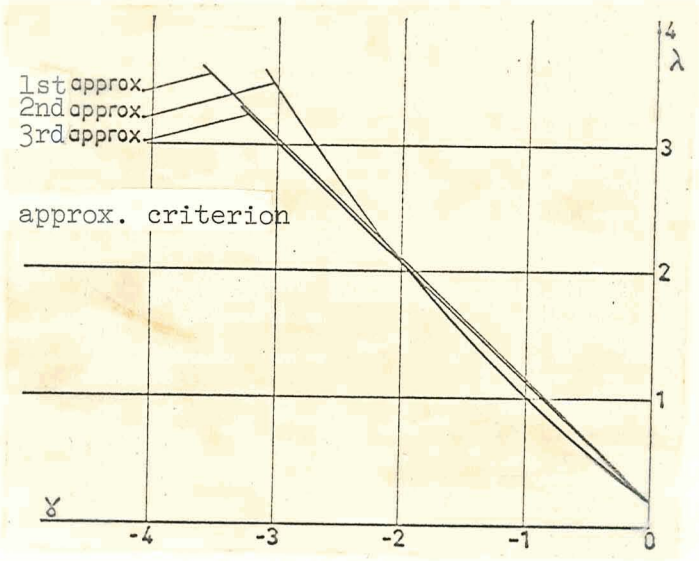

Figure 1

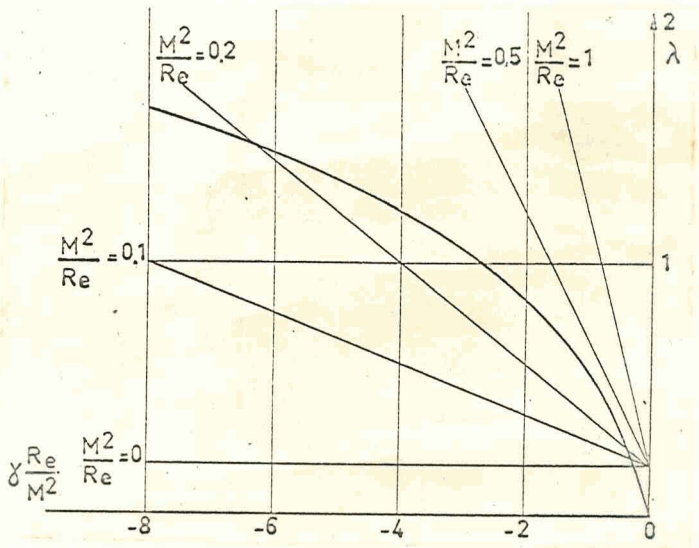

Figure 2

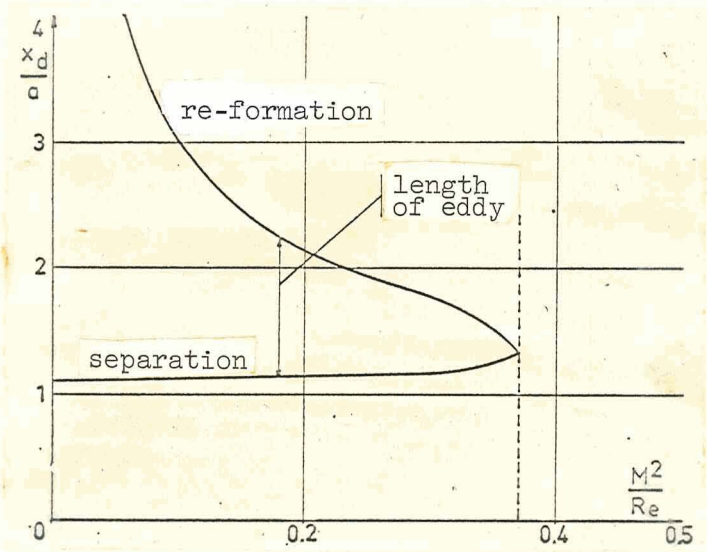

Figure 3

	Exterior velocity	Representative curves on the plane	Conditions for existence of separation	Approximate position
1.....	$\frac{u_0}{U}=1-\frac{x}{a}$	$\lambda=-\frac{M^2}{Re}\gamma$	$\frac{M^2}{Re}<1$	$\lambda_d=\frac{0.2}{1-\frac{M^2}{Re}}$
2.....	$\frac{u_0}{U}=2\sin\theta$	$\lambda=-\frac{2\cos\theta}{1+\cos\theta}$; $\gamma=-\frac{M^2}{Re}\frac{1}{1+\cos\theta}$	$\frac{M^2}{Re}<2$	$\cos\theta_d=-\frac{\frac{M^2}{Re}+0.2}{2.2}$
3.....	$\frac{u_0}{U}=\frac{a}{x}$	$\gamma=-\frac{M^2}{Re}\lambda e^{\lambda}$	$\frac{M^2}{Re}<0.37$	

In the three cases, a fairly intense magnetic field may eliminate separation. For example, case No. 3 of our calculation is based on the existence of a point of re-formation of the boundary layer on the downstream side of the separation, i.e., of a constant eddy bound to the sheet. Its length and position may be evaluated from Figure 3. /4

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Reference

1. Meksyn. New Methods in Laminar Boundary Layer Theory, Pergamon Press, 1961.

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